A Comparative Study of a Fuzzy Similarity Measure Aggregated with Fuzzy Implications Applied to Shape Recognition

Leila Baccour, Wafa Maghrebi, Slim Kanoun and Mohamed Adel Alimi

National School of Engineering, University of Sfax, Tunisia
leila.baccour@ieee.org, wafa.maghrebi@fsegs.rnu.tn,
slim.kanoun@enis.rnu.tn, adel.alimi@enis.rnu.tn

Abstract. The choice of a similarity measure is very important to compare between two samples. In addition, the similarity between many fuzzy sets (i.e., many linguistic variables) needs aggregation operators which can influence the fuzzy similarity results. In this paper we compare results of fuzzy implications aggregated to a fuzzy similarity measure applied to shapes recognition which are described by an extended curvature scale space (CSS) descriptor. We present experimental results on the vision Speech and Signal Processing Surrey University database.

Key words: Fuzzy similarity measure, Fuzzy implication, Shape recognition

1 Introduction

In several system of recognition, clustering, classification, etc. a similarity measure is used. The similarity of two samples is often evaluated by measuring a distance between their features. Thus, two samples are not considered similar if the difference between their sets of features is obvious.
Selecting the suitable distance or similarity is not a trivial task. Our purpose is to use fuzzy similarity measure to compare between shapes. A fuzzy similarity measures scale the equality degree between two fuzzy sets. In literature, many fuzzy similarity measures were presented and discussed e.g. [1, 2]. In this paper, we aggregate a fuzzy similarity measure with fuzzy implications to recognize shapes. We present and compare results for each aggregation.
In section 2, we present the description of the shape database which is followed by the construction of correspondent fuzzy database. In section 4, we give an overview of some fuzzy similarity measures from literature and some operations on fuzzy sets. Thus, we present a comparative study of experimental results of similarity measures aggregate with fuzzy operators.

2 Database Shape Presentation

We use the Vision, Speech and Signal Processing Surrey University database, which contains about 1100 images of marine creatures. Each image shows one

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distinct species on a uniform background. Every image is processed to recover the boundary edge, which is then represented by an extended curvature scale space (CSS) descriptors.

2.1 Curvature Scale Space Descriptors

Introduced by Mokhtarian et al [3, 4], the CSS descriptors register the concavities of a curve as it goes through successive filtering. The role of filtering is to smooth out the curve and gradually eliminate concavities of increasing size. More precisely, given a form described by its normalized planar contour curve.

$$\Gamma(u) = \{x(u), y(u) \,|u \in [0,1]\} \ . \tag{1}$$

The curvature at any point u is defined as the tangent angle to the curve and is computed as:

$$k(u) = \frac{x_u(u)\, y_{uu}(u) - x_{uu}(u)\, y_u(u)}{\left(x_u(u)^2 + y_u(u)^2\right)^{\frac{3}{2}}} \ . \tag{2}$$

To compute its CSS descriptors, a curve is repeatedly smoothed out using a Gaussian kernel g(u,σ). The contour of the filtered curve is represented as:

$$\Gamma(u) = \{x(u,\sigma), y(u,\sigma) \,|u \in [0,1]\} \ . \tag{3}$$

where, x(u,σ) and y(u,σ) represent the result of convolving x(u) and y(u) with g(u,σ), respectively. The curvature k(u,σ) of the smoothed out curve is represented as:

$$k(u,\sigma) = \frac{x_u(u,\sigma)\, y_{uu}(u,\sigma) - x_{uu}(u,\sigma)\, y_u(u,\sigma)}{\left(x_u(u,\sigma)^2 + y_u(u,\sigma)^2\right)^{\frac{3}{2}}} \ . \tag{4}$$

The main idea behind CSS descriptors is to extract inflection points of a curve at different values of σ. As σ increases, the evolving shape of the curve becomes smoother and we notice a progressive disappearance of the concave parts of the shape until we end up with a completely convex form (Fig. 1.). Using the curve's multi-scale representation, we can locate the points of inflection at each scale (i.e. points where k(u,σ) = 0). A graph, called CSS image, specifying the location u of these inflection points vs. the value of σ can be created:

$$I(u,\sigma) = \{(u,\sigma) \,|k(u,\sigma) = 0\} \ . \tag{5}$$

Figure 2 shows the CSS image corresponding to the shape shown in Fig. 1. Different peaks present in the CSS image correspond to the major concave segments of the shape. The maxima of the peaks are extracted and used to describe the input shape. Even though the CSS descriptors have the advantage of being invariant to scale, translation, and rotation, and are shown to be robust and tolerant of noise, they are inadequate to represent the convex segments of a shape. In addition, the CSS descriptors can be considered as local features and hence do not capture the global shape of an image contour. The following section presents a remedy (the extended CSS descriptors) for these drawbacks.

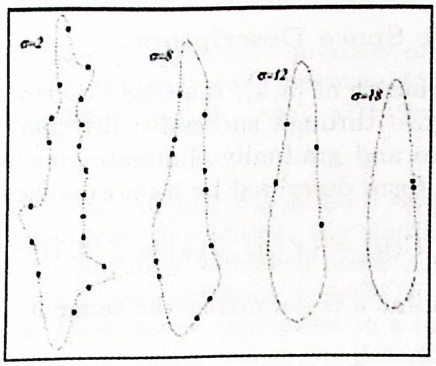


Fig. 1. Evolution of the form according to the smoothing scale σ

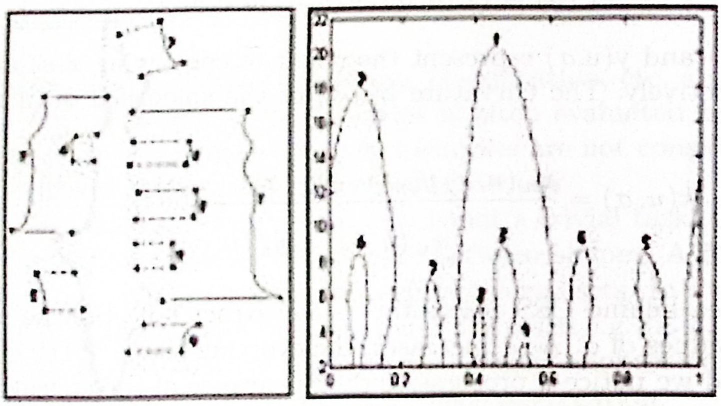


Fig. 2. Creating the CSS image of a shape

2.2 Extended CSS Descriptors

Kopf et al [5] presented a solution to remedy the inability of the CSS descriptors to represent convex segments of a shape. The idea they proposed is to create a dual shape of the input shape where all convex segments are transformed to concave segments. The dual shape is created by mirroring the input shape with respect to the circle of minimum radius R that encloses the original shape (Fig. 3.). More precisely, each point (x(u),y(u)) of the original shape is paired with a point (x'(u),y'(u)) of the dual shape such as the distance from (x(u),y(u)) to the circle is the same from (x'(u),y'(u)) to the circle. The coordinates of the circle's centre O(M_x, M_y) are calculated as:

$$M_x = \frac{1}{N}\sum_{u=1}^{N} x(u) \ . \tag{6}$$

$$M_y = \frac{1}{N}\sum_{u=1}^{N} y(u) \ . \tag{7}$$

The projected point (x'(u),y'(u)) is located at

$$x'(u) = \frac{2R - D_{x(u),y(y)}}{D_{x(u),y(y)}}(x(u) - M_x) + M_x \ . \tag{8}$$

$$y'(u) = \frac{2R - D_{x(u),y(u)}}{D_{x(u),y(u)}}(y(u) - M_y) + M_y \ . \tag{9}$$

where, $D_{x(u),y(u)}$ is the distance between the circle's centre and the original shape pixel.
Since CSS descriptors as considered local features, we decided to use two extra global features: circularity and eccentricity. Circularity is defined as:

$$cir = \frac{p^2}{A} \ . \tag{10}$$

where, P is the perimeter of the shape and A is its area. Eccentricity is defined as:

$$ecc = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} \ . \tag{11}$$

where, λ_{max} and λ_{min} are the eigenvalues of the matrix A

$$A = \begin{bmatrix} \mu_{2,0} & \mu_{1,1} \\ \mu_{1,1} & \mu_{0,2} \end{bmatrix} \ . \tag{12}$$

$\mu_{2,0}$, $\mu_{1,1}$, and $\mu_{0,2}$ are the central moments of the shape defined as:

$$\mu_{p,q} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q \ . \tag{13}$$

with $\bar{x}$ and $\bar{y}$ representing the coordinates of the shape's centroid. The eccentricity feature is size, rotation and translation invariant.
The extended CSS descriptors we used in the image indexing are a combination of four sets of features:

- Circularity feature (global feature)
- Eccentricity feature (global feature)
- CSS descriptors of original shape (local features)
- CSS descriptors of dual shape (local features)

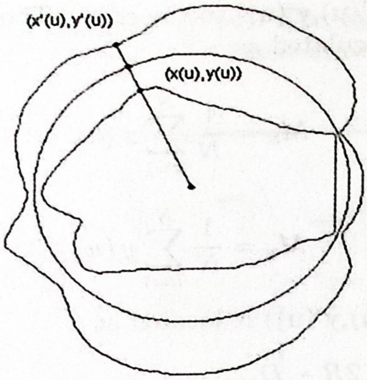


Fig. 3. Creating a dual shape with respect to an enclosing circle

3 Construction of Fuzzy Shapes Database

In this section, we are interesting to create a fuzzy shapes database. This stage permits to pass from real database constructed by real values for extended CSS descriptors to a fuzzy database. This last data are values between 0 and 1, appointing membership degrees to fuzzy sets [6]. Thus, every feature of shapes is represented with fuzzy sets which design fuzzy membership function.
To choose the membership functions, we divide database shape on two databases: the first constituted of 734 shapes serves as training database and the second constituted of 366 shapes serves as test database. Every shape is represented by the extended CSS descriptor defined in second section.
To represent fuzzy sets of concavity and convexity we think that abscises are not important to retain for matching. Because, every shape can take different positions, so two shapes can be similar if concavity ordinates are equal independently of abscises (i.e. one abscises can be on the right, other on the left). So, we retain for each shape: eccentricity, circularity, ordinate concavity points and ordinates convexity points.
We find that values of concavity, convexity and circularity can be represented by three fuzzy sets: low medium and high. However, the eccentricity is also represented by two fuzzy sets: low and high.
We choose trapezoidal function to represent fuzzy sets. We present below (Fig. 4.) the fuzzy sets for the feature convexity.

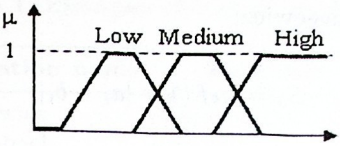


Fig. 4. Convexity membership functions

The real values of CSS descriptors of each shape are fuzzified by calculating the membership degrees to correspondent functions.

4 Overview of Some Fuzzy Similarity Measures

Many fuzzy similarity measures are presented in literature; in the sequel we present some of them.
Let $U = \{x_1, x_2, \ldots, x_n\}$ a discourse universe, A and B tow fuzzy sets in the universe of discourse defined as follow:
$A = \{(x, \mu_A(x)) | x \in U, \mu_A(x) \in [0, 1]\}$, $B = \{(x, \mu_B(x)) | x \in U, \mu_B(x) \in [0, 1]\}$
Suppose a and b the vector representations of the fuzzy sets A and B defined as a=$(a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_n)$, $a_i, b_i \in [0, 1]$, $i = 1, 2, \ldots, n$.

- Measures proposed in [7]

$$S_1 = 1 - \sum_{i=1}^{n} \frac{|a_i - b_i|}{n} . \tag{14}$$

$$S_2 = \max_i (\min(a_i, b_i)) . \tag{15}$$

$$S_3 = \frac{a.b}{\max(a.a, b.b)} . \tag{16}$$

- Measures proposed in [8]

$$S_4 = \frac{\sum_{i=1}^{n} \frac{min(a_i,b_i)}{max(a_i,b_i)}}{n} . \tag{17}$$

with the convention $\frac{0}{0} = 1$

$$S_5 = \sum_{i=1}^{n} \frac{1 - |a_i - b_i|}{n} . \tag{18}$$

- Measures proposed in [9]

fuzzy similarity of Lukasiewiez:

$$S_6 = inf\,(1 - |a_i - b_i|) \quad . \tag{19}$$

Fuzzy similarity of Gödel:

$$S_7 = inf\,(a_i \leftrightarrow_{Go} b_i) \quad . \tag{20}$$

where

$$a_i \leftrightarrow_{Go} b_i = \begin{cases} 1 \text{ if } & a_i = b_i \\ min\,(a_i, b_i) \text{ else} \end{cases}$$

Fuzzy similarity of Goguen:

$$S_8 = inf\,(a_i \leftrightarrow_G b_i) \quad . \tag{21}$$

where $a_i \leftrightarrow_G b_i = \begin{cases} 1 & \text{if } a_i = b_i \\ \frac{min(a_i,b_i)}{max(a_i,b_i)} & \text{else} \end{cases}$

5 Opertations on Fuzy Sets

As for classical sets, operations are defined on fuzzy sets, such as intersection, union, complement, etc. In the literature on fuzzy sets, a large number of possible definitions are proposed to implement intersection, union and complement. The operators defined by Zadeh are:

- Intersection operator: $min(\mu_A(x), \mu_B(x))$
- Union operator: $max(\mu_A(x), \mu_B(x))$

Where A and B are fuzzy sets defined in precedent section.
General forms of intersection and union are represented by triangular norms (T-norms) and (T-conorms) and designs fuzzy implications.
A fuzzy implication is a function from $[0, 1] \times [0, 1]$ to [0,1] which determine the verity degree of the preposition $x \Rightarrow y$. Table 1 gives examples of implications used most often [10, 11].

Table 1. Examples of fuzzy implications

Implication name	Function
Lukasiewicz	min(1-x+y,1)
Reichenbach	1-x+xy
Goguen	$\begin{cases} min(\frac{x}{y},1) \text{ if } & x \neq 0 \\ 1\ else \end{cases}$
Kleene-Dienes	max (1-x, y)
Brouwer-Gödel	$\begin{cases} 1 \text{ if } & x \leq y \\ y\ else \end{cases}$
Rescher-Gaines	$\begin{cases} 1 \text{ if } & x \leq y \\ 0\ else \end{cases}$
Willmott	max (1-x, min(x,y))

6 Similarity Measures Based on Fuzzy Logic

Our objective is to measure the similarity of two shapes having fuzzified attributes according to fuzzy sets of features. Let shapes A and B be described by M linguistic variables v_i and for each linguistic variable v_i, linguistic values are defined L_k^i. Each linguistic value L_k^i is represented by a fuzzy set with a membership function $\mu_{L_k^i}$. The similarity of A and B is computed on two steps:

- Compute similarity of A and B according to one linguistic value L_k^i for all variables v_i, $S_{L_k^i}(A, B)$
- Compute similarity of A and B according to all lingistic values for all variables, S(A,B)

We obtain similarity measures $S_{L_k^i}(A, B)$ of A and B according to every linguistic value L_k^i for all variables v_i. Thus, we must find an aggregation or a relation between similarity degrees $S_{L_k^i}(A, B)$ to find the similarity S(A,B) of two shapes. So, we use implication operators to compute this similarity. We choose the similarity measurement S_6 defined in section 4 (equation (19)) to compute $S_{L_k^i}(A, B)$. Then, we aggregate the results to implication operators defined in section 5 and we compare results.

7 Experimental Results

We trial similarity measure S_6 aggregated to each implication and we compare results of matching two images from test database to 100 images of training set database (Fig. 5. and Fig. 6.). The images retrieved with the highest similarity rates (i.e., degree of similarity equal 0.5 or higher) shown in Fig. 5. b and Fig. 6. b obtained, using S_6 aggregated to Gödel implication and the images retrieved with the highest similarity ranks shown in Fig. 5. c and Fig. 6. c obtained, using S_6 aggregated to Willmott implication.

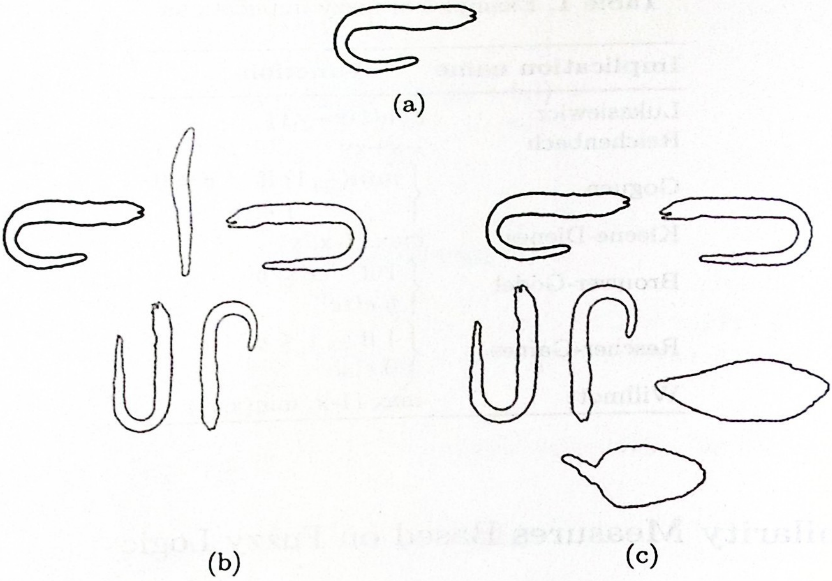


Fig. 5. (a) Target image (b) matching images using S_6 and Gödel implication (c) matching images using S_6 and Willmott implication

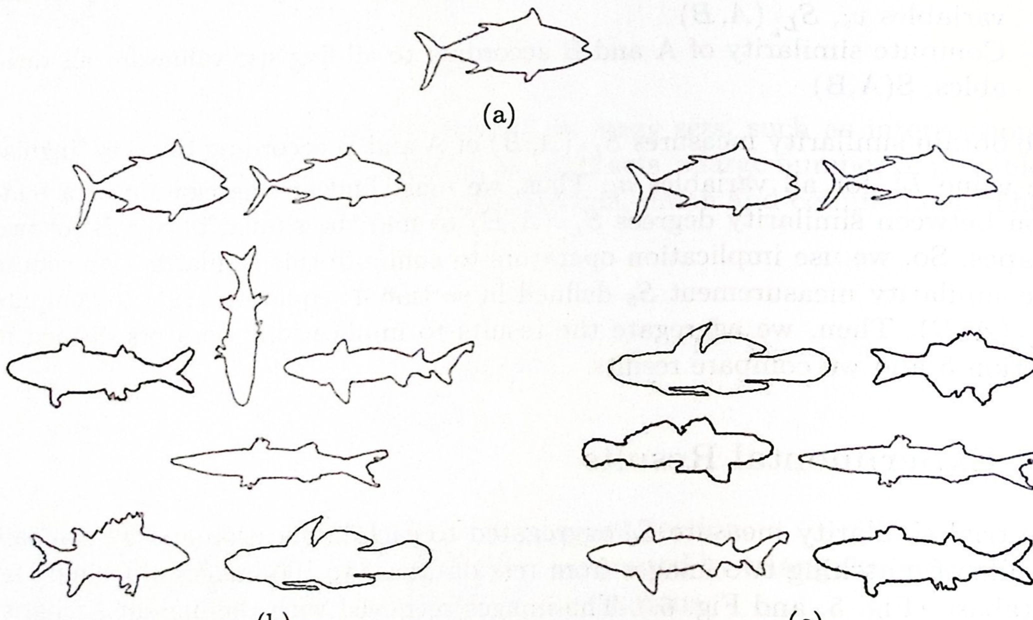
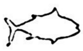


Fig. 6. (a) Target image (b) matching images using S_6 and Gödel implication (c) first's matching images using S_6 and Willmott implication

If we examine Fig. 5. and Fig. 6., we remark that the results produced with S_6 and Gödel implication are different from the results produced with S_6 and Willmot implication. To find the implication aggregated with S_6 which produces best results, we compute recall rate and precision rate that we present in table 2.

Table 2. Results obtained for each implication aggregated with S_6

Implication	Shape	Recall rate (%)	Precision rate (%)
Lukasiewicz		60%	75%
		57.14%	72.72%
Reichenbach		57,14%	100%
		61,53%	72.72%
Goguen		**80%**	**100%**
		61,53%	72.72%
Kleene-Dienes		57.14%	100%
		61,53%	72.72%
Gödel		**80%**	**100%**
		**87,5%**	63.63%
Rescher-Gaines		**80%**	**100%**
		**87,5%**	63,63%
Willmott		66,66%	100%
		75%	54,54%

We can observe from the table 2 that the best results are produced by S_6 aggregated with Goguen, Gödel or Rescher-Gaines implications.

8 Conclusion

Eventually, we presented a comparative study of aggregation implications to a fuzzy similarity applied to shape recognition. The experimental results show the difference between the implications aggregation. So, for the same shape sample, we found for the aggregation of Gödel implication to a fuzzy similarity a recall rate equal to 80% and a precision rate equal to 100% and for the aggregation of the Willmott implication to the fuzzy similarity a recall rate equal to 66,66% and a precision rate equal to 100%. Thus, we show with this comparison the importance of the choice of the aggregation operator in order to compute the fuzzy similarity between many linguistic variables.

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